IES
Intelligent Energy Systems

12 May 2003 Issue 002

insider

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Applying Black-Scholes to Electricity Markets

Overview

The applicability of the Black-Scholes derivative pricing approach to electricity markets has been an issue since the start of the NEM. This article briefly reviews the basis of the model and the circumstances in which it can be used in electricity markets, particularly the NEM.

In the early 1970s, Fisher Black and Myron Scholes made a major breakthrough by developing an option-pricing formula. The original Black-Scholes option-pricing model was developed to value options, primarily on equities. Now the Black-Scholes model is frequently used as a standard pricing tool in the stock and commodity markets, including electricity.

In deriving the option formula, Black and Scholes assumed that the underlying asset price follows a geometric Brownian motion with constant volatility. A typical example of geometric Brownian motion movement is shown in Figure 1.

Figure 1 Stock Price of Commonwealth Bank from 1/1/1996 to 31/12/2002

From the geometric Brownian motion equation, the European Call and Put option[1] formulas can be derived respectively as:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2) \qquad (1)$$

and

[1] A Call/Put option gives the holder the right to buy/sell the underlying asset (e.g. electricity) by a certain date for a certain price. The price in the contract is known as the exercise price; the date in the contract is known as the expiration date or maturity. European options can be exercised only on the expiration date.

$$P = Ke^{-rT}N(-d_2) - S_0N(-d_1) \qquad (2)$$

where

$$d_1 = \frac{\ln\left(S_0 / K\right) + \left(r + \sigma^2 / 2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(S_0 / K\right) + \left(r - \sigma^2 / 2\right)T}{\sigma\sqrt{T}}$$

$$= d_1 - \sigma\sqrt{T}$$

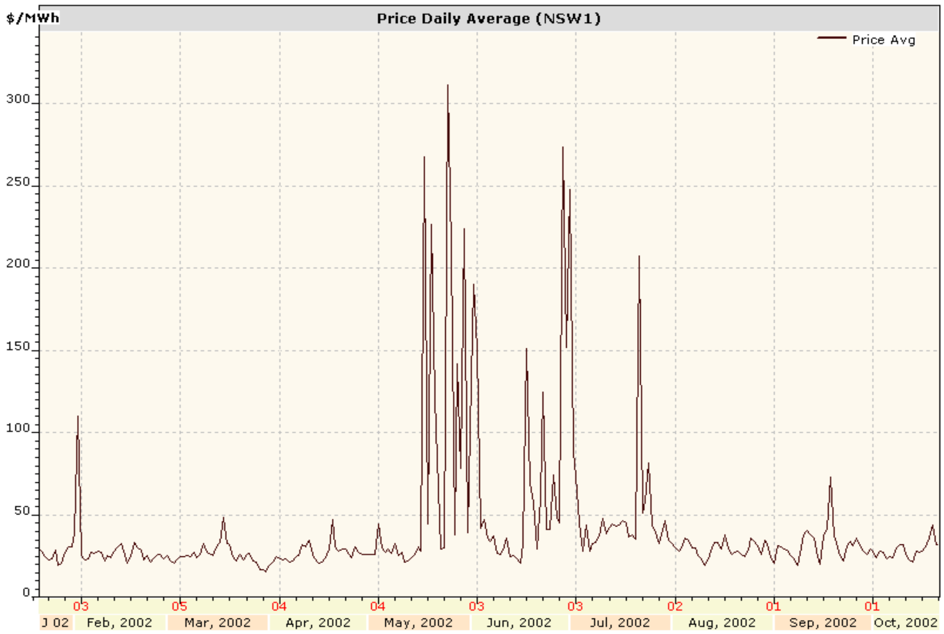
and $N(\cdot)$ represents the cumulative probability normal distribution function with a mean of zero and a standard deviation of one. Here S_0 is the commodity price at the current time, K is the strike price, r is a constant risk-free rate, T is the time to maturity of the option and σ is the volatility of commodity prices.

Electricity Spot Price

Electricity markets are often composed of two markets—the spot market where electricity is bought and sold at spot price, and a contract (or derivatives) market where contracts such as 'over the counter' and SFE are traded.

A fundamental issue to using Black-Scholes to value options on electricity commodities is whether or not the underlying price movement follows a geometric Brownian motion as the model assumes, and what electricity commodities might follow this model.

Figure 2 Example of Spot Prices from NEMCO



Electricity spot prices certainly do not follow a geometric Brownian motion as evidenced in the example of spot prices shown in Figure 2, which shows NSW spot prices for the period January to October 2002.

As observed, the characteristics of spot prices are described by the following characteristics:

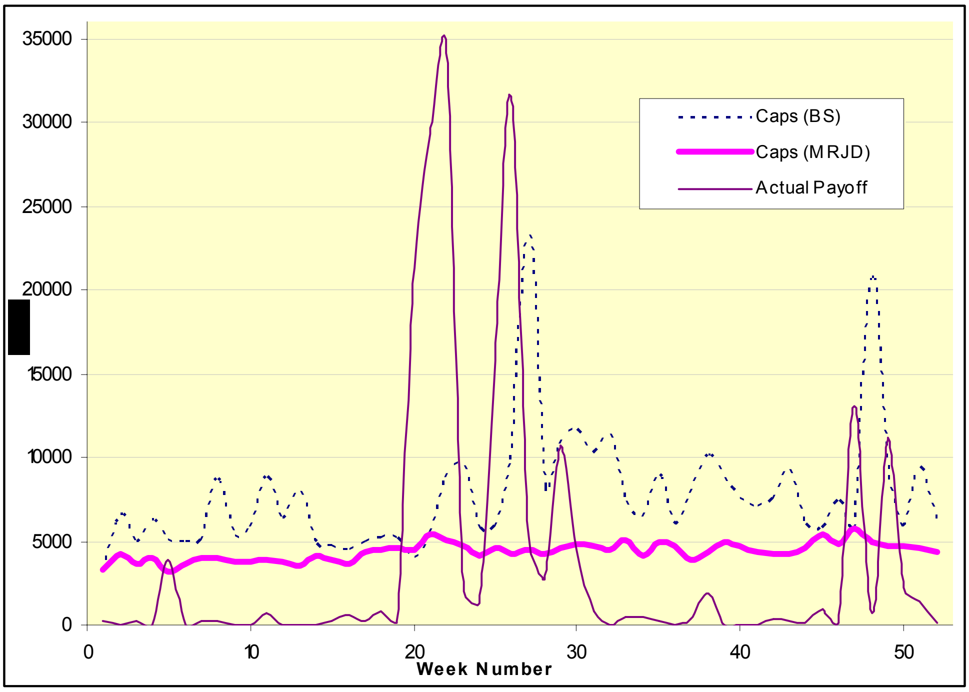
- Mean-reversion;
- Multi-seasonality: yearly, monthly, weekly, and daily;
- High Spikes.

As the geometric Brownian motion model does not have a mean-reverting feature, the Black-Scholes formula will misprice electricity option contract values if used for such valuations.

For pricing contracts referenced to the electricity spot price, other models such as a mean-reverting jump-diffusion (MRJD) model may be more appropriate, since the MRJD model can be made to more readily accommodate the characteristics of spot prices.

This is illustrated in Figure 3, which shows weekly cap prices for NSW in 2002 with a strike price of $50 using the Black-Scholes and MRJD models along with the real pay off. The Black-Scholes formula appears to overprice the contract value based on the average cap prices for the 52-week period, because of the mismatch of electricity price movement with a geometric Brownian motion model. The geometric Brownian motion model assumes that the best estimate of future prices is based on the current price and the spot price has an increasing trend which is related to interest rates. These assumptions are clearly not the case for electricity.

Figure 3 Weekly Cap Prices with the strike price $50 for NSW in 2002 (52 weeks)



The mean-reverting jump-diffusion (MRJD) model provides a better pricing framework. Of note is that the accuracy of the pricing results depends on parameter estimation from the historical data collected from the market. Also of note is that as a statistical model, the MRJD model cannot incorporate market changes in the way a structural simulation approach does.

Swaptions

There are electricity contracts where the underlying process might follow a Brownian motion with constant volatility model. One such contract is an option on a forward or futures contract. This form of contract is generally known as a swaption. The optionality is exercised on a contract as opposed to being exercised on the physical commodity itself. Thus we could consider using the Black-Scholes formula to price swaptions, since the forward and futures price might follow the geometric Brownian motion.

Table 1 shows some examples of European-style Call and Put swaptions respectively, priced using the Black-Scholes formula.

Table 1 NSW Swaption Prices valued at 6/5/2003 and expired at 31/12/2003 for the swap contract of 2004

Option Type	K=30	K=33	K=35	K=37	K=40
Call	$5.5469	$2.7569	$1.2862	$0.4286	$0.0404
Put	$0.0024	$0.1179	$0.5843	$1.6637	$4.1811

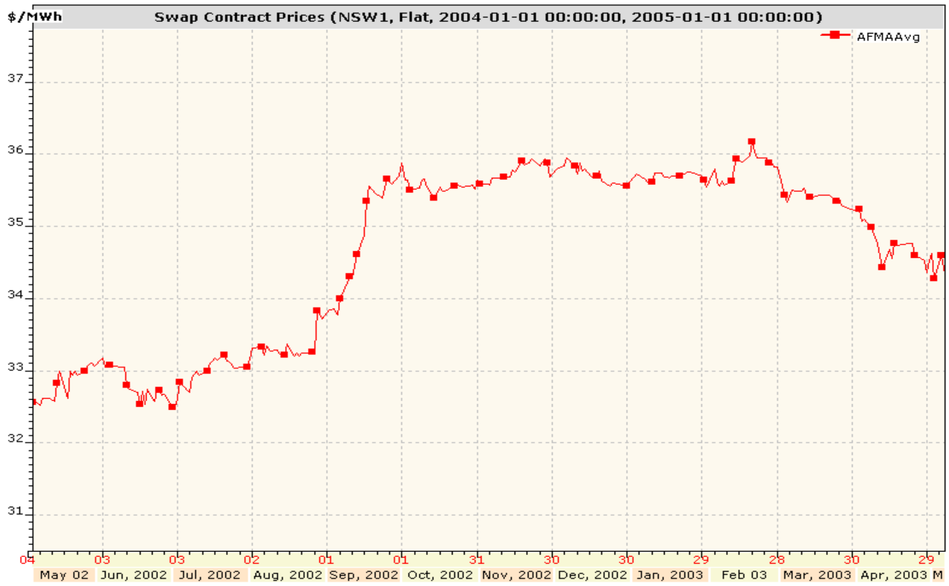
The pricing results in Table 1 are calculated from Equations (1) and (2). The parameters applied are listed as follows:

- Δt represents the time in years. Assuming that there are 245 trading days in a year in NSW, then $\Delta t = 1/245$.
- T is the time to maturity in years. Assuming that in NSW there are 165 trading days from 5/5/2003 to 31/12/2003, then $T = 165 \times \sqrt{\Delta t} = 0.673469388$
- σ represents the volatility. It is calculated from the formula:

  $$\sigma = \frac{stdev}{\sqrt{\Delta t}}$$

  where $stdev$ is the standard deviation of historical forward price data such as that supplied by AFMA and Δt is as defined above. In this example the standard derivation for the volatility is estimated from AFMA data for a flat contract in 2004, starting from 6/5/2002 and ending on 5/5/2003, giving $\sigma = 7.9538\%$.
- r is the risk free rate. The risk-free rate is referred to the current cash rate used by the Reserve Bank of Australia. The term cash rate is used to denote the interest rate which financial institutions pay to borrow or charge to lend funds in the money market on an overnight basis. In this example we have used a rate of $r = 4.75\%$.
- S_0 is the commodity price. In this example, it is the average of bids and offers for a swap contract for 2004 based on AFMA swap data for 5/5/2003, giving $S_0 = \$34.6$.
- K represents the strike price. We have chosen example values as shown in Table 1.
- $N(\cdot)$ is a normal distribution function. It is available in most statistical tools, for example pnrom in SPLUS and normdist in EXCEL.

Figure 4 **Average swap contract prices of flat rate, 2004 for NSW from 6/5/2002 to 5/5/2003**



Conclusion

The Black-Scholes model can be used to price swaptions, which are options written on forwards or futures rather than the physical commodity itself. However, the model is not appropriate for pricing caps and floors and other forms of options directly related to the electricity spot price (the electricity commodity itself).

Using NEO to price contracts

Modules that incorporate the Black-Scholes and MRJD models are about to be made available in NEO. NEO has access to the data required for these models including AFMA data.

For information on these modules contact Ron de Jong at IES Sydney or Andrew Campbell at IES Melbourne.

A look at Some Recent Regional Supply Curves

One of the most convenient ways of considering and understanding the profile of generator bids is to view regional and corporation generator supply curves which plot offer quantities versus price. Issues that can be of interest include:

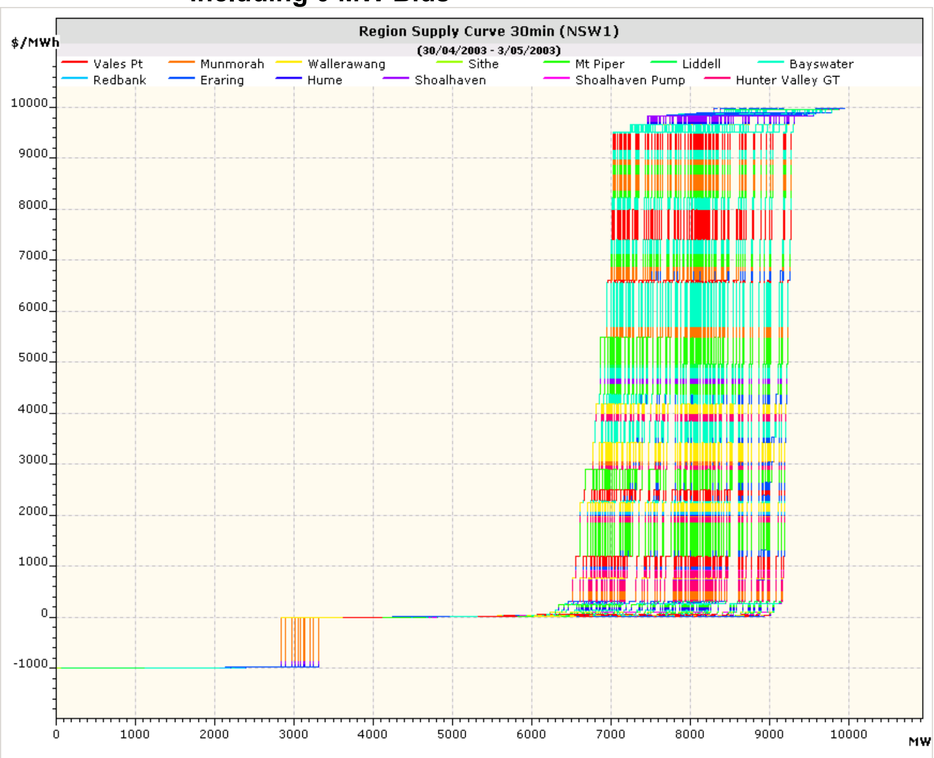
- How much these supply curves vary during the day, week, month, year;
- What is their general shape;
- What can one ascertain from these profiles in terms of contract positions and general trading strategies.

The IES product NEO (version 3) now provides for supply curves to be viewed in a number of ways. These include supply curves for individual corporations and total regional supply curves. Supply curves can be plotted as a single supply curve for a specified time, or all the individual half hourly supply curves for a specified time interval can be superimposed on the same graph.

The case we examine here presents all the half hourly supply curves for the region of NSW for the first three days in May this year (Thursday 1 May 2003 to Saturday 3 May 2003, plotted on the same graph. This representation has the advantage of seeing how the supply curves vary over the nominated period. Points to note on the supply curves shown include:

- The supply curves are plotted with and without 0 MW bids. Although bands of 0 MW bid are not used, they remain available for use, and hence provide important information.
- Each bid band represents a MW volume and price. The discontinuous ‘bid steps’ are plotted, as is conventional, with a vertical line joining the last step to the next. The vertical line combined with the horizontal line that represents the bid quantity, form a bid step. In the regional supply curves shown here, the bids from the various corporations are assigned different colours.

Figure 5 Half hourly supply curves for NSW, 1 May to 3 May 2003, including 0 MW Bids



The number of bands bid in at 0 MW is highlighted by the difference between Figure 5, which shows the supply curves with the 0MW bids, and Figure 6, which shows the supply curves without the 0MW bids.

Figure 6 Half hourly supply curves for NSW, 1 May to 3 May 2003, excluding 0 MW bids

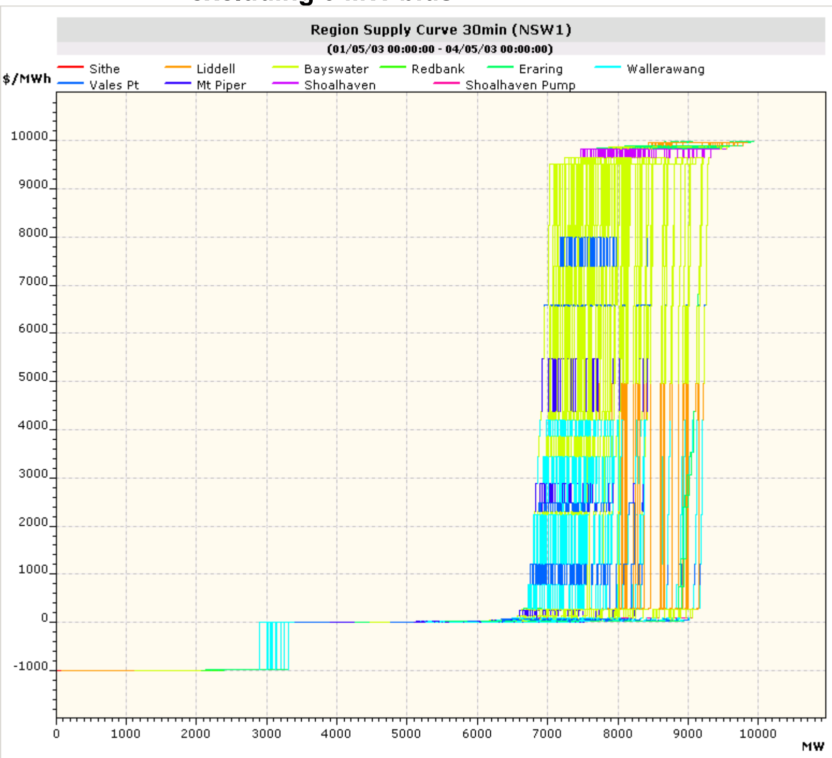
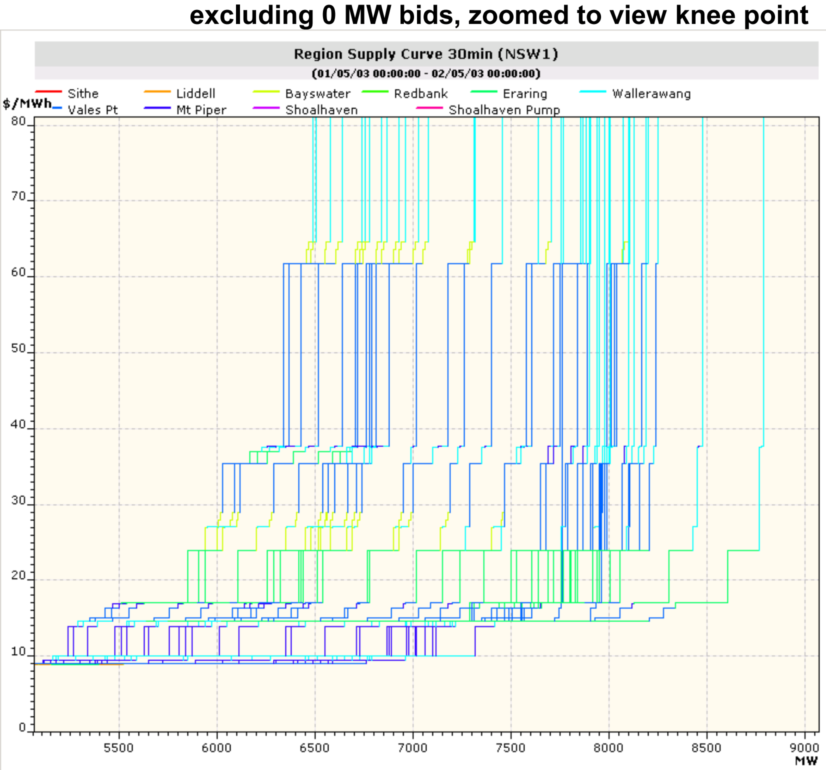


Figure 7 shows the supply curves presented in Figure 6 'zoomed in' at the 'knee point'. Of particular note is that the supply curves maintain their shape and that the interleaving of corporation bids is unchanged. The slope becomes very steep at prices greater than about $90/MWh.

Figure 7 Half hourly supply curves for NSW, 1 May 2003, excluding 0 MW bids, zoomed to view knee point



Supply curves for Queensland for Thursday 1 May 2003, excluding the 0 MW bid bands, are shown in Figure 8 (the whole curve) and Figure 9 (zoomed in around the knee point).

Figure 8 Half hourly supply curves for Queensland, excluding 0 MW Bids, Thursday 1 May 2003

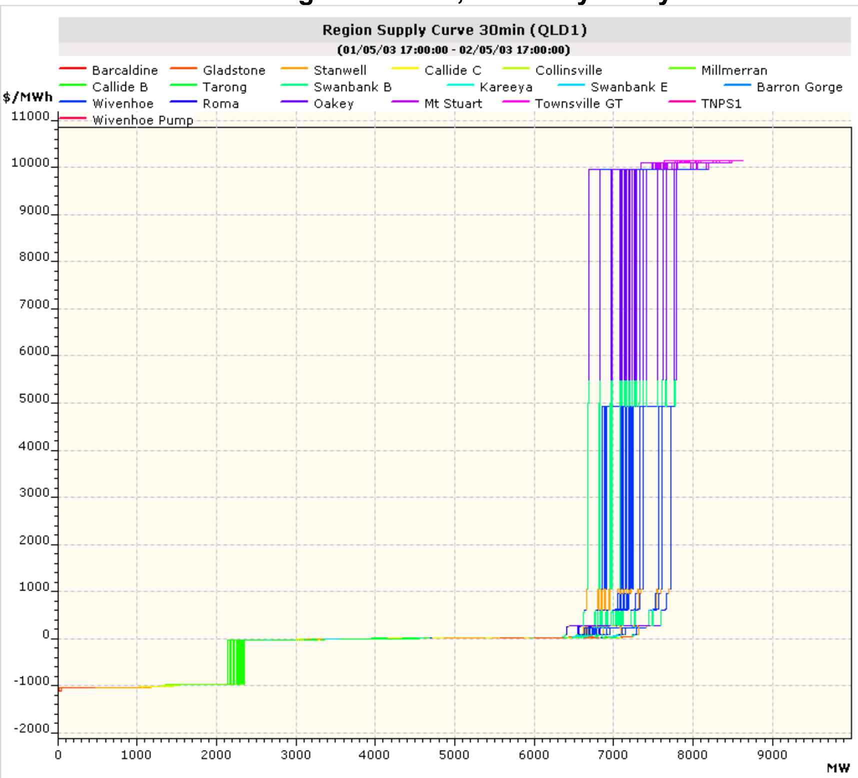
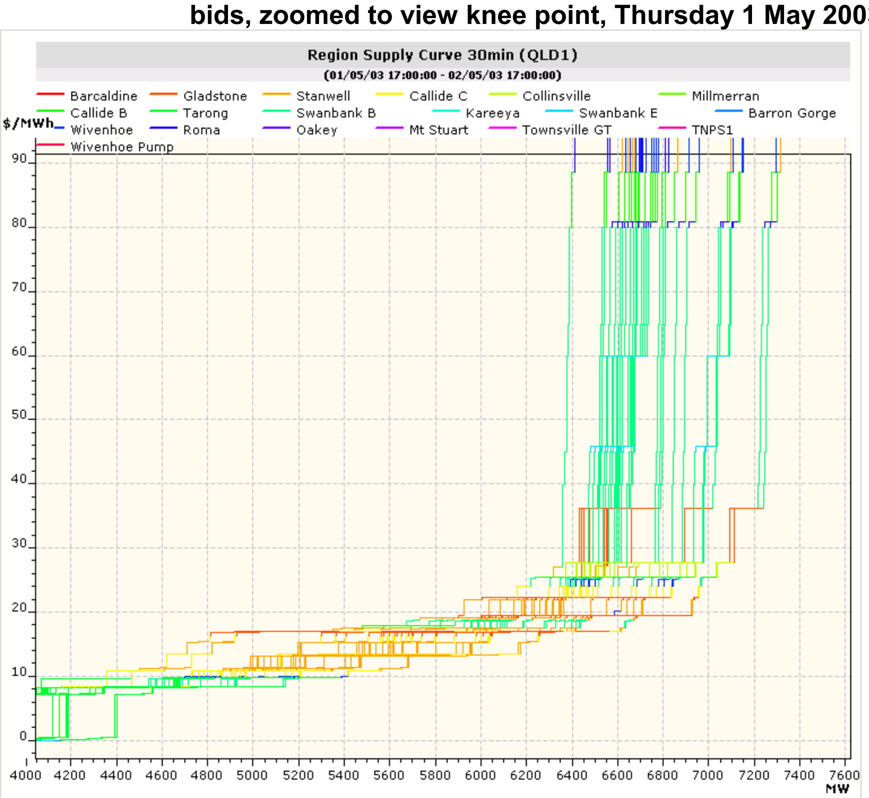


Figure 9 Half-hourly supply curves for Queensland, excluding 0 MW bids, zoomed to view knee point, Thursday 1 May 2003



As shown in the preceding graphs, NEO 3 now has the ability to present supply curves for any specified period in a number of new ways. These new presentations are very useful to ascertain aspects of trading not evident in other presentations. To find out more, contact Andrew Campbell at IES Melbourne or Ron de Jong at IES Sydney.

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